# NMR Relaxation in Multipolar AMX Systems under Spin-Locking Conditions

Andrei Kaikkonen and Jozef Kowalewski

*Division of Physical Chemistry, Arrhenius Laboratory, University of Stockholm, S-10691 Stockholm, Sweden*

A relaxation network has been calculated for multipolar AMX systems under application of a spin-locking RF field. Systems of this type are of interest in the study of proteins with fractional $^2$H enrichment. All possible auto- and cross-correlation terms involving dipolar, quadrupolar, and CSA interactions have been taken into account. The results show the presence of spectral densities at zero frequency for interactions associated with the locked nuclei, which are nonvanishing in the absence of fast motions. In addition, the application of a spin-locking field blocks certain cross-correlation interactions, thereby considerably simplifying the relaxation network.   © 2000 Academic Press

## INTRODUCTION

Nuclear magnetic relaxation has proved to be a very valuable tool in the study of molecular dynamics (*1, 2*). NMR relaxation effects are also a primary source of structural information (*3, 4*). Among the most widely used experiments for studying biological macromolecules are NOESY (*5*) and its counterpart under application of a radiofrequency (RF) field, ROESY (*6, 7*). The disadvantage of the former is that cross peaks vanish within a certain motional regime, whereas ROESY is free from this problem. Both NOESY and ROESY experiments rely on the effect of cross relaxation between different spins. The intensities of cross peaks reflect the strength of homonuclear or heteronuclear dipolar interactions, thus providing information on the molecular environment. In the presence of quadrupolar (*8–10*) and CSA (*11*) interactions, however, the situation becomes rather complicated, involving various auto- and cross-correlation processes.

Investigations of the proteins by means of liquid-state NMR are presently restricted to molecular sizes of about 30 kDa, using $^{13}$C and $^{15}$N heteronuclear spectroscopy. One way to push this boundary upward is to combine such experiments with fractional $^2$H enrichment (*12, 13*). And, since deuterium nuclei have spin = 1, quadrupolar interactions now come into play. Typically, one obtains a heteronuclear spin system consisting of $^1$H, $^{13}$C (or $^{15}$N), and $^2$H nuclei, and therefore systems of this type are considered in the present paper. Moreover, this is the simplest spin system that simultaneously involves dipolar–dipolar, dipolar–quadrupolar, dipolar–CSA, and quadrupolar–

CSA cross-correlations. We are therefore interested in seeing how these cross-correlations are blocked upon application of a spin-locking field on the nuclei.

Relaxation dynamics with and without application of an RF field in homonuclear and heteronuclear multispin 1/2 systems have been considered by various authors (*14–21*). There are two main advantages of applying RF fields in these systems. First, on- and off-resonance RF fields block or modify certain relaxation pathways, thus simplifying the interpretation of experiments. Second, the spin-locked relaxation contains nonvanishing contributions from slow dynamic processes, which is especially important for large biomolecules.

This paper shows the derivation of a full relaxation matrix for heteronuclear AMX systems with spins 1/2, 1/2, and 1, respectively, with and without application of an on-resonance spin-locking field. Auto- and cross-correlation terms between dipolar, quadrupolar, and CSA interactions have been taken into account. We have previously performed similar calculations for the case of multipolar AX systems (*10*). The AX spin system does not have dipolar–dipolar cross correlations, which under certain spin locking condition, may compete with strong quadrupolar relaxation mechanism in AMX spin system as will be shown later.

## THEORY

The evolution of the density operator, $\sigma$, in the presence of relaxation is described by (*22*)

$$\frac{d\sigma}{dt} = -i[H, \sigma] + R(\sigma - \sigma^{eq}), \qquad [1]$$

where $H$ is the static Hamiltonian, $\sigma^{eq}$ is the equilibrium density operator, and $R$ is the relaxation superoperator. The operator responsible for relaxation can be recast as spin and lattice parts

$$F = \sum_{\eta, q} (-1)^q T_q^\eta V_q^\eta, \qquad [2]$$

### TABLE 1
### Nonzero Longitudinal Relaxation Matrix Elements

$$R_{1,1} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + \frac{8}{9} J^D_{IS}(\omega_I - \omega_S) + J^D_{IL}(\omega_I) + \frac{8}{3} J^D_{IS}(\omega_I) + 2J^D_{IL}(\omega_I + \omega_L) + \frac{16}{3} J^D_{IS}(\omega_I + \omega_S) + 4J^C_I(\omega_I)$$

$$R_{2,2} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + \frac{8}{9} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_L) + \frac{8}{3} J^D_{LS}(\omega_L) + 2J^D_{IL}(\omega_I + \omega_L) + \frac{16}{3} J^D_{LS}(\omega_L + \omega_S) + 4J^C_L(\omega_L)$$

$$R_{3,3} = \frac{1}{3} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{3} J^D_{LS}(\omega_L - \omega_S) + J^D_{IS}(\omega_S) + J^D_{LS}(\omega_S) + 2J^D_{IS}(\omega_I + \omega_S) + 2J^D_{LS}(\omega_L + \omega_S) + 4J^Q_S(\omega_S) + 16J^Q_S(2\omega_S) + 4J^C_S(\omega_S)$$

$$R_{4,4} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + \frac{1}{9} J^D_{IS}(\omega_I - \omega_S) + J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_I) + \frac{4}{3} J^D_{IS}(\omega_I) + 3J^D_{IS}(\omega_S) + 3J^D_{LS}(\omega_S) + 2J^D_{IL}(\omega_I + \omega_L)$$

$$+ \frac{2}{3} J^D_{IS}(\omega_I + \omega_S) + 6J^D_{LS}(\omega_L + \omega_S) + 12J^Q_S(\omega_S) + 4J^C_I(\omega_I) + 12J^C_S(\omega_S)$$

$$R_{5,5} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + J^D_{IS}(\omega_I - \omega_S) + \frac{1}{9} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_L) + \frac{4}{3} J^D_{LS}(\omega_L) + 3J^D_{IS}(\omega_S) + 3J^D_{LS}(\omega_S) + 2J^D_{IL}(\omega_I + \omega_L)$$

$$+ 6J^D_{IS}(\omega_I + \omega_S) + \frac{2}{3} J^D_{LS}(\omega_L + \omega_S) + 12J^Q_S(\omega_S) + 4J^C_L(\omega_L) + 12J^C_S(\omega_S)$$

$$R_{6,6} = \frac{1}{3} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{3} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_I) + J^D_{IL}(\omega_L) + 4J^D_{IS}(\omega_I) + J^D_{IS}(\omega_S) + 4J^D_{LS}(\omega_L) + J^D_{LS}(\omega_S) + 2J^D_{IS}(\omega_I + \omega_S)$$

$$+ 2J^D_{LS}(\omega_L + \omega_S) + 4J^Q_S(\omega_S) + 16J^Q_S(2\omega_S) + 4J^C_I(\omega_I) + 4J^C_L(\omega_L) + 4J^C_S(\omega_S)$$

$$R_{7,7} = \frac{8}{9} J^D_{IS}(\omega_I - \omega_S) + \frac{8}{9} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_I) + J^D_{IL}(\omega_L) + \frac{8}{3} J^D_{IS}(\omega_I) + \frac{8}{3} J^D_{LS}(\omega_L) + \frac{16}{3} J^D_{IS}(\omega_I + \omega_S) + \frac{16}{3} J^D_{LS}(\omega_L + \omega_S)$$

$$+ 4J^C_I(\omega_I) + 4J^C_L(\omega_L)$$

$$R_{8,8} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + \frac{1}{3} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{3} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_I) + 4J^D_{IS}(\omega_I) + J^D_{IS}(\omega_S) + J^D_{LS}(\omega_S) + 2J^D_{IL}(\omega_I + \omega_L)$$

$$+ 2J^D_{IS}(\omega_I + \omega_S) + 2J^D_{LS}(\omega_L + \omega_S) + 4J^Q_S(\omega_S) + 16J^Q_S(2\omega_S) + 4J^C_I(\omega_I) + 4J^C_S(\omega_S)$$

$$R_{9,9} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + \frac{1}{3} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{3} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_L) + J^D_{IS}(\omega_S) + 4J^D_{LS}(\omega_L) + J^D_{LS}(\omega_S) + 2J^D_{IL}(\omega_I + \omega_L)$$

$$+ 2J^D_{IS}(\omega_I + \omega_S) + 2J^D_{LS}(\omega_L + \omega_S) + 4J^Q_S(\omega_S) + 16J^Q_S(2\omega_S) + 4J^C_L(\omega_L) + 4J^C_S(\omega_S)$$

$$R_{10,10} = J^D_{IS}(\omega_I - \omega_S) + J^D_{LS}(\omega_L - \omega_S) + 3J^D_{IS}(\omega_S) + 3J^D_{LS}(\omega_S) + 6J^D_{IS}(\omega_I + \omega_S) + 6J^D_{LS}(\omega_L + \omega_S) + 12J^Q_S(\omega_S) + 12J^C_S(\omega_S)$$

$$R_{11,11} = \frac{1}{9} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{9} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_I) + J^D_{IL}(\omega_L) + \frac{4}{3} J^D_{IS}(\omega_I) + 3J^D_{IS}(\omega_S) + \frac{4}{3} J^D_{LS}(\omega_L) + 3J^D_{LS}(\omega_S)$$

$$+ \frac{2}{3} J^D_{IS}(\omega_I + \omega_S) + \frac{2}{3} J^D_{LS}(\omega_L + \omega_S) + 12J^Q_S(\omega_S) + 4J^C_I(\omega_I) + 4J^C_L(\omega_L) + 12J^C_S(\omega_S)$$

$$R_{1,2} = -\frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + 2J^D_{IL}(\omega_I + \omega_L)$$

$$R_{1,3} = -\frac{4}{3\sqrt{6}} J^D_{IS}(\omega_I - \omega_S) + \frac{8}{\sqrt{6}} J^D_{IS}(\omega_I + \omega_S)$$

$$R_{1,4} = -\frac{4}{9\sqrt{2}} J^D_{IS}(\omega_I - \omega_S) + \frac{8}{3\sqrt{2}} J^D_{IS}(\omega_I) - \frac{8}{3\sqrt{2}} J^D_{IS}(\omega_I + \omega_S)$$

$$R_{1,6} = 4\sqrt{\frac{2}{3}} K^{D,D}_{IL,IS}(\omega_I)$$

$$R_{2,3} = -\frac{4}{3\sqrt{6}} J^D_{LS}(\omega_L - \omega_S) + \frac{8}{\sqrt{6}} J^D_{LS}(\omega_L + \omega_S)$$

$$R_{2,5} = -\frac{4}{9\sqrt{2}} J^D_{LS}(\omega_L - \omega_S) + \frac{8}{3\sqrt{2}} J^D_{LS}(\omega_L) - \frac{8}{3\sqrt{2}} J^D_{LS}(\omega_L + \omega_S)$$

$$R_{2,6} = 4\sqrt{\frac{2}{3}} K^{D,D}_{IL,LS}(\omega_L)$$

$$R_{3,4} = \frac{1}{3\sqrt{3}} J^D_{IS}(\omega_I - \omega_S) - \frac{2}{\sqrt{3}} J^D_{IS}(\omega_I + \omega_S) - \frac{12}{\sqrt{3}} K^{D,Q}_{IS,S}(\omega_S)$$

$$R_{3,5} = \frac{1}{3\sqrt{3}} J^D_{LS}(\omega_L - \omega_S) - \frac{2}{\sqrt{3}} J^D_{LS}(\omega_L + \omega_S) - \frac{12}{\sqrt{3}} K^{D,Q}_{LS,S}(\omega_S)$$

$$R_{3,6} = 2K^{D,D}_{IS,LS}(\omega_S)$$

**TABLE 1**—*Continued*

$$R_{4,5} = -\frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + 2J_{IL}^{D}(\omega_I + \omega_L) + 6K_{IS,LS}^{D,D}(\omega_S)$$

$$R_{4,6} = -\frac{1}{\sqrt{3}} J_{LS}^{D}(\omega_L - \omega_S) + \frac{6}{\sqrt{3}} J_{LS}^{D}(\omega_L + \omega_S) + \frac{4}{\sqrt{3}} K_{IL,IS}^{D,D}(\omega_I) - \frac{12}{\sqrt{3}} K_{LS,S}^{D,Q}(\omega_S)$$

$$R_{5,6} = -\frac{1}{\sqrt{3}} J_{IS}^{D}(\omega_I - \omega_S) + \frac{6}{\sqrt{3}} J_{IS}^{D}(\omega_I + \omega_S) + \frac{4}{\sqrt{3}} K_{IL,LS}^{D,D}(\omega_L) - \frac{12}{\sqrt{3}} K_{IS,S}^{D,Q}(\omega_S)$$

$$R_{7,8} = -\frac{4}{3\sqrt{6}} J_{LS}^{D}(\omega_L - \omega_S) + \frac{8}{\sqrt{6}} J_{LS}^{D}(\omega_L + \omega_S) + 4\sqrt{\frac{2}{3}} K_{IL,IS}^{D,D}(\omega_I)$$

$$R_{7,9} = -\frac{4}{3\sqrt{6}} J_{IS}^{D}(\omega_I - \omega_S) + \frac{8}{\sqrt{6}} J_{IS}^{D}(\omega_I + \omega_S) + 4\sqrt{\frac{2}{3}} K_{IL,LS}^{D,D}(\omega_L)$$

$$R_{7,11} = -\frac{2\sqrt{2}}{9} J_{IS}^{D}(\omega_I - \omega_S) - \frac{2\sqrt{2}}{9} J_{LS}^{D}(\omega_L - \omega_S) + \frac{4\sqrt{2}}{3} J_{IS}^{D}(\omega_I) + \frac{4\sqrt{2}}{3} J_{LS}^{D}(\omega_L)$$

$$- \frac{4\sqrt{2}}{3} J_{IS}^{D}(\omega_I + \omega_S) - \frac{4\sqrt{2}}{3} J_{LS}^{D}(\omega_L + \omega_S)$$

$$R_{8,9} = -\frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + 2J_{IL}^{D}(\omega_I + \omega_L) + 2K_{IS,LS}^{D,D}(\omega_S)$$

$$R_{8,10} = -\frac{1}{\sqrt{3}} J_{IS}^{D}(\omega_I - \omega_S) + \frac{6}{\sqrt{3}} J_{IS}^{D}(\omega_I + \omega_S) - \frac{12}{\sqrt{3}} K_{IS,S}^{D,Q}(\omega_S)$$

$$R_{8,11} = \frac{1}{3\sqrt{3}} J_{LS}^{D}(\omega_L - \omega_S) - \frac{2}{\sqrt{3}} J_{LS}^{D}(\omega_L + \omega_S) + \frac{4}{\sqrt{3}} K_{IL,IS}^{D,D}(\omega_I) - \frac{12}{\sqrt{3}} K_{LS,S}^{D,Q}(\omega_S)$$

$$R_{9,10} = -\frac{1}{\sqrt{3}} J_{LS}^{D}(\omega_L - \omega_S) + \frac{6}{\sqrt{3}} J_{LS}^{D}(\omega_L + \omega_S) - \frac{12}{\sqrt{3}} K_{LS,S}^{D,Q}(\omega_S)$$

$$R_{9,11} = \frac{1}{3\sqrt{3}} J_{IS}^{D}(\omega_I - \omega_S) - \frac{2}{\sqrt{3}} J_{IS}^{D}(\omega_I + \omega_S) + \frac{4}{\sqrt{3}} K_{IL,LS}^{D,D}(\omega_L) - \frac{12}{\sqrt{3}} K_{IS,S}^{D,Q}(\omega_S)$$

$$R_{10,11} = 6K_{IS,LS}^{D,D}(\omega_S)$$

$$R_{1,7} = -4K_{IL,I}^{D,C}(\omega_I)$$

$$R_{1,8} = -\frac{16}{\sqrt{6}} K_{IS,I}^{D,C}(\omega_I)$$

$$R_{2,7} = -4K_{IL,L}^{D,C}(\omega_L)$$

$$R_{2,9} = -\frac{16}{\sqrt{6}} K_{LS,L}^{D,C}(\omega_L)$$

$$R_{3,8} = -4K_{IS,S}^{D,C}(\omega_S)$$

$$R_{3,9} = -4K_{LS,S}^{D,C}(\omega_S)$$

$$R_{3,10} = \frac{24}{\sqrt{3}} K_{S,S}^{Q,C}(\omega_S)$$

$$R_{4,8} = -\frac{8}{\sqrt{3}} K_{IS,I}^{D,C}(\omega_I) + \frac{24}{\sqrt{3}} K_{S,S}^{Q,C}(\omega_S)$$

$$R_{4,10} = -12K_{IS,S}^{D,C}(\omega_S)$$

$$R_{4,11} = -4K_{IL,I}^{D,C}(\omega_I) - 12K_{LS,S}^{D,C}(\omega_S)$$

$$R_{5,9} = -\frac{8}{\sqrt{3}} K_{LS,L}^{D,C}(\omega_L) + \frac{24}{\sqrt{3}} K_{S,S}^{Q,C}(\omega_S)$$

$$R_{5,10} = -12K_{LS,S}^{D,C}(\omega_S)$$

$$R_{5,11} = -4K_{IL,L}^{D,C}(\omega_L) - 12K_{IS,S}^{D,C}(\omega_S)$$

$$R_{6,7} = -8\sqrt{\frac{2}{3}} K_{IS,I}^{D,C}(\omega_I) - 8\sqrt{\frac{2}{3}} K_{LS,L}^{D,C}(\omega_L)$$

$$R_{6,8} = -4K_{IL,I}^{D,C}(\omega_I) - 4K_{LS,S}^{D,C}(\omega_S)$$

$$R_{6,9} = -4K_{IL,L}^{D,C}(\omega_L) - 4K_{IS,S}^{D,C}(\omega_S)$$

$$R_{6,11} = -\frac{8}{\sqrt{3}} K_{IS,I}^{D,C}(\omega_I) - \frac{8}{\sqrt{3}} K_{LS,L}^{D,C}(\omega_L) + \frac{24}{\sqrt{3}} K_{S,S}^{Q,C}(\omega_S)$$

**TABLE 2**
**Nonzero Relaxation Matrix Elements with the Spin-1/2 Nucleus *I* Spin-Locked**

$$R^I_{1,1} = \frac{2}{3} J^D_{IL}(0) + \frac{16}{9} J^D_{IS}(0) + \frac{8}{3} J^C_I(0) + \frac{3}{18} J^D_{IL}(\omega_I - \omega_L) + \frac{4}{9} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{2} J^D_{IL}(\omega_I) + J^D_{IL}(\omega_L) + \frac{4}{3} J^D_{IS}(\omega_I)$$

$$+ \frac{8}{3} J^D_{IS}(\omega_S) + J^D_{IL}(\omega_I + \omega_L) + \frac{8}{3} J^D_{IS}(\omega_I + \omega_S) + 2 J^C_I(\omega_I)$$

$$R^I_{2,2} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + \frac{8}{9} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_L) + \frac{8}{3} J^D_{LS}(\omega_L) + 2 J^D_{IL}(\omega_I + \omega_L) + \frac{16}{3} J^D_{LS}(\omega_L + \omega_S) + 4 J^C_L(\omega_L)$$

$$R^I_{3,3} = \frac{1}{3} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{3} J^D_{LS}(\omega_L - \omega_S) + J^D_{IS}(\omega_S) + J^D_{LS}(\omega_S) + 2 J^D_{IS}(\omega_I + \omega_S) + 2 J^D_{LS}(\omega_L + \omega_S) + 4 J^Q_S(\omega_S) + 16 J^Q_S(2\omega_S) + 4 J^C_S(\omega_S)$$

$$R^I_{4,4} = \frac{2}{3} J^D_{IL}(0) + \frac{8}{9} J^D_{IS}(0) + \frac{8}{3} J^C_I(0) + \frac{3}{18} J^D_{IL}(\omega_I - \omega_L) + \frac{5}{9} J^D_{IS}(\omega_I - \omega_S) + J^D_{LS}(\omega_L - \omega_S) + \frac{1}{2} J^D_{IL}(\omega_I) + J^D_{IL}(\omega_L) + \frac{2}{3} J^D_{IS}(\omega_I)$$

$$+ \frac{1}{3} J^D_{IS}(\omega_S) + 3 J^D_{LS}(\omega_S) + J^D_{IL}(\omega_I + \omega_L) + \frac{10}{3} J^D_{IS}(\omega_I + \omega_S) + 6 J^D_{LS}(\omega_L + \omega_S) + 12 J^Q_S(\omega_S) + 2 J^C_I(\omega_I) + 12 J^C_S(\omega_S)$$

$$R^I_{5,5} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + J^D_{IS}(\omega_I - \omega_S) + \frac{1}{9} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_L) + \frac{4}{3} J^D_{LS}(\omega_L) + 3 J^D_{IS}(\omega_S) + 3 J^D_{LS}(\omega_S) + 2 J^D_{IL}(\omega_I + \omega_L)$$

$$+ 6 J^D_{IS}(\omega_I + \omega_S) + \frac{2}{3} J^D_{LS}(\omega_L + \omega_S) + 12 J^Q_S(\omega_S) + 4 J^C_L(\omega_L) + 12 J^C_S(\omega_S)$$

$$R^I_{6,6} = \frac{2}{3} J^D_{IL}(0) + \frac{8}{3} J^D_{IS}(0) + \frac{8}{3} J^C_I(0) + \frac{1}{6} J^D_{IL}(\omega_I - \omega_L) + \frac{1}{3} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{3} J^D_{LS}(\omega_L - \omega_S) + \frac{1}{2} J^D_{IL}(\omega_I) + 2 J^D_{IS}(\omega_I) + J^D_{IS}(\omega_S)$$

$$+ 4 J^D_{LS}(\omega_L) + J^D_{LS}(\omega_S) + J^D_{IL}(\omega_I + \omega_L) + 2 J^D_{IS}(\omega_I + \omega_S) + 2 J^D_{LS}(\omega_L + \omega_S) + 4 J^Q_S(\omega_S) + 16 J^Q_S(2\omega_S) + 2 J^C_I(\omega_I) + 4 J^C_L(\omega_L) + 4 J^C_S(\omega_S)$$

$$R^I_{7,7} = \frac{2}{3} J^D_{IL}(0) + \frac{16}{9} J^D_{IS}(0) + \frac{8}{3} J^C_1(0) + \frac{1}{6} J^D_{IL}(\omega_I - \omega_L) + \frac{4}{9} J^D_{IS}(\omega_I - \omega_S) + \frac{8}{9} J^D_{LS}(\omega_L - \omega_S) + \frac{1}{2} J^D_{IL}(\omega_I) + \frac{4}{3} J^D_{IS}(\omega_I)$$

$$+ \frac{8}{3} J^D_{IS}(\omega_S) + \frac{8}{3} J^D_{LS}(\omega_L) + J^D_{IL}(\omega_I + \omega_L) + \frac{8}{3} J^D_{IS}(\omega_I + \omega_S) + \frac{16}{3} J^D_{LS}(\omega_L + \omega_S) + 2 J^C_I(\omega_I) + 4 J^C_L(\omega_L)$$

$$R^I_{8,8} = \frac{2}{3} J^D_{IL}(0) + \frac{8}{3} J^D_{IS}(0) + \frac{8}{3} J^C_I(0) + \frac{1}{6} J^D_{IL}(\omega_I - \omega_L) + \frac{1}{3} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{3} J^D_{LS}(\omega_L - \omega_S) + \frac{1}{2} J^D_{IL}(\omega_I) + J^D_{IL}(\omega_L) + 2 J^D_{IS}(\omega_I)$$

$$+ J^D_{IS}(\omega_S) + J^D_{LS}(\omega_S) + J^D_{IL}(\omega_I + \omega_L) + 2 J^D_{IS}(\omega_I + \omega_S) + 2 J^D_{LS}(\omega_L + \omega_S) + 4 J^Q_S(\omega_S) + 16 J^Q_S(2\omega_S) + 2 J^C_I(\omega_I) + 4 J^C_S(\omega_S)$$

$$R^I_{9,9} = \frac{1}{3} J^D_{IL}(\omega_I - \omega_L) + \frac{1}{3} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{3} J^D_{LS}(\omega_L - \omega_S) + J^D_{IL}(\omega_L) + J^D_{IS}(\omega_S) + 4 J^D_{LS}(\omega_L) + J^D_{LS}(\omega_S) + 2 J^D_{IL}(\omega_I + \omega_L)$$

$$+ 2 J^D_{IS}(\omega_I + \omega_S) + 2 J^D_{LS}(\omega_L + \omega_S) + 4 J^Q_S(\omega_S) + 16 J^Q_S(2\omega_S) + 4 J^C_L(\omega_L) + 4 J^C_S(\omega_S)$$

$$R^I_{10,10} = J^D_{IS}(\omega_I - \omega_S) + J^D_{LS}(\omega_L - \omega_S) + 3 J^D_{IS}(\omega_S) + 3 J^D_{LS}(\omega_S) + 6 J^D_{IS}(\omega_I + \omega_S) + 6 J^D_{LS}(\omega_L + \omega_S) + 12 J^Q_S(\omega_S) + 12 J^C_S(\omega_S)$$

$$R^I_{11,11} = \frac{2}{3} J^D_{IL}(0) + \frac{8}{9} J^D_{IS}(0) + \frac{8}{3} J^C_I(0) + \frac{1}{6} J^D_{IL}(\omega_I - \omega_L) + \frac{5}{9} J^D_{IS}(\omega_I - \omega_S) + \frac{1}{9} J^D_{LS}(\omega_L - \omega_S) + \frac{1}{2} J^D_{IL}(\omega_I) + \frac{2}{3} J^D_{IS}(\omega_I) + \frac{1}{3} J^D_{IS}(\omega_S)$$

$$+ \frac{4}{3} J^D_{LS}(\omega_L) + 3 J^D_{LS}(\omega_S) + J^D_{IL}(\omega_I + \omega_L) + \frac{10}{3} J^D_{IS}(\omega_I + \omega_S) + \frac{2}{3} J^D_{LS}(\omega_L + \omega_S) + 12 J^Q_S(\omega_S) + 2 J^C_I(\omega_I) + 4 J^C_L(\omega_L) + 12 J^C_S(\omega_S)$$

$$R^I_{1,4} = \frac{16}{9\sqrt{2}} J^D_{IS}(0) - \frac{2}{9\sqrt{2}} J^D_{IS}(\omega_I - \omega_S) + \frac{4}{3\sqrt{2}} J^D_{IS}(\omega_I) - \frac{4}{3\sqrt{2}} J^D_{IS}(\omega_S) - \frac{4}{3\sqrt{2}} J^D_{IS}(\omega_I + \omega_S)$$

$$R^I_{1,6} = \frac{8}{3} \sqrt{\frac{2}{3}} K^{D,D}_{IL,IS}(0) + 2\sqrt{\frac{2}{3}} K^{D,D}_{IL,IS}(\omega_I)$$

$$R^I_{2,3} = -\frac{4}{3\sqrt{6}} J^D_{LS}(\omega_L - \omega_S) + \frac{8}{\sqrt{6}} J^D_{LS}(\omega_L + \omega_S)$$

$$R^I_{2,5} = -\frac{4}{9\sqrt{2}} J^D_{LS}(\omega_L - \omega_S) + \frac{8}{3\sqrt{2}} J^D_{LS}(\omega_L) - \frac{8}{3\sqrt{2}} J^D_{LS}(\omega_L + \omega_S)$$

$$R^I_{3,5} = \frac{1}{3\sqrt{3}} J^D_{LS}(\omega_L - \omega_S) - \frac{2}{\sqrt{3}} J^D_{LS}(\omega_L + \omega_S) - \frac{12}{\sqrt{3}} K^{D,Q}_{LS,S}(\omega_S)$$

$$R^I_{4,6} = \frac{8}{3\sqrt{3}} K^{D,D}_{IL,IS}(0) - \frac{1}{\sqrt{3}} J^D_{LS}(\omega_L - \omega_S) + \frac{6}{\sqrt{3}} J^D_{LS}(\omega_L + \omega_S) + \frac{2}{\sqrt{3}} K^{D,D}_{IL,IS}(\omega_I) - \frac{12}{\sqrt{3}} K^{D,Q}_{LS,S}(\omega_S)$$

$$R^I_{7,8} = \frac{8}{3} \sqrt{\frac{2}{3}} K^{D,D}_{IL,IS}(0) - \frac{4}{3\sqrt{6}} J^D_{LS}(\omega_L - \omega_S) + \frac{8}{\sqrt{6}} J^D_{LS}(\omega_L + \omega_S) + 2\sqrt{\frac{2}{3}} K^{D,D}_{IL,IS}(\omega_I)$$

TABLE 2—*Continued*

$$R^I_{7,11} = \frac{8\sqrt{2}}{9} J^D_{IS}(0) - \frac{\sqrt{2}}{9} J^D_{IS}(\omega_I - \omega_S) - \frac{2\sqrt{2}}{9} J^D_{LS}(\omega_L - \omega_S) + \frac{2\sqrt{2}}{3} J^D_{IS}(\omega_I) - \frac{2\sqrt{2}}{3} J^D_{IS}(\omega_S)$$

$$+ \frac{4\sqrt{2}}{3} J^D_{LS}(\omega_L) - \frac{2\sqrt{2}}{3} J^D_{IS}(\omega_I + \omega_S) - \frac{4\sqrt{2}}{3} J^D_{LS}(\omega_L + \omega_S)$$

$$R^I_{8,11} = \frac{8}{3\sqrt{3}} K^{D,D}_{IL,IS}(0) + \frac{1}{3\sqrt{3}} J^D_{LS}(\omega_L - \omega_S) - \frac{2}{\sqrt{3}} J^D_{LS}(\omega_L + \omega_S) + \frac{2}{\sqrt{3}} K^{D,D}_{IL,IS}(\omega_I) - \frac{12}{\sqrt{3}} K^{D,Q}_{LS,S}(\omega_S)$$

$$R^I_{9,10} = -\frac{1}{\sqrt{3}} J^D_{LS}(\omega_L - \omega_S) + \frac{6}{\sqrt{3}} J^D_{LS}(\omega_L + \omega_S) - \frac{12}{\sqrt{3}} K^{D,Q}_{LS,S}(\omega_S)$$

$$R^I_{1,7} = -\frac{8}{3} K^{D,C}_{IL,I}(0) - 2K^{D,C}_{IL,I}(\omega_I)$$

$$R^I_{1,8} = -\frac{32}{3\sqrt{6}} K^{D,C}_{IS,I}(0) - \frac{8}{\sqrt{6}} K^{D,C}_{IS,I}(\omega_I)$$

$$R^I_{2,9} = -\frac{16}{\sqrt{6}} K^{D,C}_{LS,L}(\omega_L)$$

$$R^I_{3,9} = -4K^{D,C}_{LS,S}(\omega_S)$$

$$R^I_{3,10} = -\frac{24}{\sqrt{3}} K^{Q,C}_{S,S}(\omega_S)$$

$$R^I_{4,8} = -\frac{16}{3\sqrt{3}} K^{D,C}_{IS,I}(0) - \frac{4}{\sqrt{3}} K^{D,C}_{IS,I}(\omega_I) + \frac{24}{\sqrt{3}} K^{Q,C}_{S,S}(\omega_S)$$

$$R^I_{4,11} = -\frac{8}{3} K^{D,C}_{IL,I}(0) - 2K^{D,C}_{IL,I}(\omega_I) - 12K^{D,C}_{LS,S}(\omega_S)$$

$$R^I_{5,9} = -\frac{8}{\sqrt{3}} K^{D,C}_{LS,L}(\omega_L) + \frac{24}{\sqrt{3}} K^{Q,S}_{S,S}(\omega_S)$$

$$R^I_{5,10} = -12K^{D,C}_{LS,S}(\omega_S)$$

$$R^I_{6,7} = -\frac{16}{3} \sqrt{\frac{2}{3}} K^{D,C}_{IS,I}(0) - 4\sqrt{\frac{2}{3}} K^{D,C}_{IS,I}(\omega_I) - 8\sqrt{\frac{2}{3}} K^{D,C}_{LS,L}(\omega_L)$$

$$R^I_{6,8} = -\frac{8}{3} K^{D,C}_{IL,I}(0) - 2K^{D,C}_{IL,I}(\omega_I) - 4K^{D,C}_{LS,S}(\omega_S)$$

$$R^I_{6,11} = -\frac{16}{3\sqrt{3}} K^{D,C}_{IS,I}(0) - \frac{4}{\sqrt{3}} K^{D,C}_{IS,I}(\omega_I) - \frac{8}{\sqrt{3}} K^{D,C}_{LS,L}(\omega_L) + \frac{24}{\sqrt{3}} K^{Q,C}_{S,S}(\omega_S)$$

where $T^\eta_q$ and $V^\eta_q$ are the $q$th components of spin and lattice operators, respectively, and $\eta$ represents different relaxation mechanisms. The density operator can be spanned over the set of orthonormal basis operators $\nu^k$

$$\sigma = \sum_k a^k \nu^k. \qquad [3]$$

In these terms, the relaxation matrix elements are described by

$$R_{mn} = \sum_{\mu,\lambda,p,q} (-1)^q J^{\mu,\lambda}(\omega_q) tr\{\nu^m, [T^\mu_{p,-q}, [T^\lambda_{p,q}, \nu^n]]\},$$

with

$$J^{\mu,\lambda}(\omega) = \text{Re} \int_0^\infty \langle V^\mu_{-q}(t - \tau) V^\lambda_q(t)\rangle \exp(-i\omega\tau)d\tau. \qquad [4]$$

Here $p$ represents rank and $q$ order. If $\mu$ and $\lambda$ are the same, then $J^{\mu,\lambda}_k(\omega)$ is called the auto-correlation spectral density, and if $\mu$ and $\lambda$ are different, then $J^{\mu,\lambda}_k(\omega)$ is associated with cross-correlation of different interactions and will be denoted below as $K^{\mu,\lambda}_k(\omega)$. Various spectral density functions have been reviewed by Woessner (*23*).

The $T^\mu_{p,q}$ operators are time-dependent (*15*), and for the considered interactions they have the rank equal to two, which is omitted for simplicity. We assume that scalar relaxation of the second kind (*24, 25*) is not efficient. This is normally the case when spin-locking fields do not satisfy the Hartmann–Hahn condition, $\omega_{1X} \neq \omega_{1A,M}$ (*26*), or when the quadrupolar relaxation mechanism is not very efficient, as can be the case for deuterons. Scalar relaxation of the first kind and random field interactions have not been taken into account, because they are rarely important. Moreover, the latter is represented by rank-one tensors and therefore cannot give any interference effects with rank-two tensors in isotropic media. The following expressions are used for the dipolar interaction between spins $k$ and $l$

**TABLE 3**
**Nonzero Relaxation Matrix Elements with the Spin-1 Nucleus $S$ Spin-Locked**

$$R_{1,1}^{S} = \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{8}{9} J_{IS}^{D}(\omega_I - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{8}{3} J_{IS}^{D}(\omega_I) + 2J_{IL}^{D}(\omega_I + \omega_L) + \frac{16}{3} J_{IS}^{D}(\omega_I + \omega_S) + 4J_I^{C}(\omega_I)$$

$$R_{2,2}^{S} = \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{8}{9} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_L) + \frac{8}{3} J_{LS}^{D}(\omega_L) + 2J_{IL}^{D}(\omega_I + \omega_L) + \frac{16}{3} J_{LS}^{D}(\omega_L + \omega_S) + 4J_L^{C}(\omega_L)$$

$$R_{3,3}^{S} = \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_S^{Q}(0) + \frac{8}{3} J_S^{C}(0) + \frac{1}{6} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{6} J_{LS}^{D}(\omega_L - \omega_S) + J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S)$$

$$+ J_{LS}^{D}(\omega_L) + \frac{1}{2} J_{LS}^{D}(\omega_S) + J_{IS}^{D}(\omega_I + \omega_S) + J_{LS}^{D}(\omega_L + \omega_S) + 10J_S^{Q}(\omega_S) + 4J_S^{Q}(2\omega_S) + 2J_S^{C}(\omega_S)$$

$$R_{4,4}^{S} = 2J_{IS}^{D}(0) + 2J_{LS}^{D}(0) + 8J_S^{C}(0) + \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{5}{18} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I)$$

$$+ \frac{1}{3} J_{IS}^{D}(\omega_I) + \frac{3}{2} J_{IS}^{D}(\omega_S) + 3J_{LS}^{D}(\omega_L) + \frac{3}{2} J_{LS}^{D}(\omega_S) + 2J_{IL}^{D}(\omega_I + \omega_L) + \frac{5}{3} J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ 3J_{LS}^{D}(\omega_L + \omega_S) + 6J_S^{Q}(\omega_S) + 6J_S^{Q}(2\omega_S) + 4J_I^{C}(\omega_I) + 6J_S^{C}(\omega_S)$$

$$R_{5,5}^{S} = 2J_{IS}^{D}(0) + 2J_{LS}^{D}(0) + 8J_S^{C}(0) + \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{2} J_{IS}^{D}(\omega_I - \omega_S) + \frac{5}{18} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_L)$$

$$+ 3J_{IS}^{D}(\omega_I) + \frac{3}{2} J_{IS}^{D}(\omega_S) + \frac{1}{3} J_{LS}^{D}(\omega_L) + \frac{3}{2} J_{LS}^{D}(\omega_S) + 2J_{IL}^{D}(\omega_I + \omega_L) + 3J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ \frac{5}{3} J_{LS}^{D}(\omega_L + \omega_S) + 6J_S^{Q}(\omega_S) + 6J_S^{Q}(2\omega_S) + 4J_L^{C}(\omega_L) + 6J_S^{C}(\omega_S)$$

$$R_{6,6}^{S} = \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_S^{Q}(0) + \frac{8}{3} J_S^{C}(0) + \frac{5}{18} J_{IS}^{D}(\omega_I - \omega_S) + \frac{5}{18} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L)$$

$$+ J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{1}{2} J_{LS}^{D}(\omega_S) + 5J_{IS}^{D}(\omega_I + \omega_S) + 5J_{LS}^{D}(\omega_L + \omega_S)$$

$$+ 10J_S^{Q}(\omega_S) + 4J_S^{Q}(2\omega_S) + 4J_I^{C}(\omega_I) + 4J_L^{C}(\omega_L) + 2J_S^{C}(\omega_S)$$

$$R_{7,7}^{S} = \frac{8}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{8}{9} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L) + \frac{8}{3} J_{IS}^{D}(\omega_I) + \frac{8}{3} J_{LS}^{D}(\omega_L) + \frac{16}{3} J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ \frac{16}{3} J_{LS}^{D}(\omega_L + \omega_S) + 4J_I^{C}(\omega_I) + 4J_L^{C}(\omega_L)$$

$$R_{8,8}^{S} = \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_S^{Q}(0) + \frac{8}{3} J_S^{C}(0) + \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{5}{6} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{6} J_{LS}^{D}(\omega_L - \omega_S)$$

$$+ J_{IL}^{D}(\omega_I) + J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{1}{2} J_{LS}^{D}(\omega_S) + 2J_{IL}^{D}(\omega_I + \omega_L) + 5J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ J_{LS}^{D}(\omega_L + \omega_S) + 10J_S^{Q}(\omega_S) + 4J_S^{Q}(2\omega_S) + 4J_I^{C}(\omega_I) + 2J_S^{C}(\omega_S)$$

$$R_{9,9}^{S} = \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_S^{Q}(0) + \frac{8}{3} J_S^{C}(0) + \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{6} J_{IS}^{D}(\omega_I - \omega_S) + \frac{5}{6} J_{LS}^{D}(\omega_L - \omega_S)$$

$$+ J_{IL}^{D}(\omega_L) + J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{1}{2} J_{LS}^{D}(\omega_S) + 2J_{IL}^{D}(\omega_I + \omega_L) + J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ 5J_{LS}^{D}(\omega_L + \omega_S) + 10J_S^{Q}(\omega_S) + 4J_S^{Q}(2\omega_S) + 4J_L^{C}(\omega_L) + 2J_S^{C}(\omega_S)$$

$$R_{10,10}^{S} = 2J_{IS}^{D}(0) + 2J_{LS}^{D}(0) + 8J_S^{C}(0) + \frac{1}{2} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + 3J_{IS}^{D}(\omega_I) + \frac{3}{2} J_{IS}^{D}(\omega_S) + 3J_{LS}^{D}(\omega_L)$$

$$+ \frac{3}{2} J_{LS}^{D}(\omega_S) + 3J_{IS}^{D}(\omega_I + \omega_S) + 3J_{LS}^{D}(\omega_L + \omega_S) + 6J_S^{Q}(\omega_S) + 6J_S^{Q}(2\omega_S) + 6J_S^{C}(\omega_S)$$

$$R_{11,11}^{S} = 2J_{IS}^{D}(0) + 2J_{LS}^{D}(0) + 8J_S^{C}(0) + \frac{5}{18} J_{IS}^{D}(\omega_I - \omega_S) + \frac{5}{18} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L) + \frac{1}{3} J_{IS}^{D}(\omega_I)$$

$$+ \frac{3}{2} J_{IS}^{D}(\omega_S) + \frac{1}{3} J_{LS}^{D}(\omega_L) + \frac{3}{2} J_{LS}^{D}(\omega_S) + \frac{5}{3} J_{IS}^{D}(\omega_I + \omega_S) + \frac{5}{3} J_{LS}^{D}(\omega_L + \omega_S)$$

$$+ 6J_S^{Q}(\omega_S) + 6J_S^{Q}(2\omega_S) + 4J_I^{C}(\omega_I) + 4J_L^{C}(\omega_L) + 6J_S^{C}(\omega_S)$$

**TABLE 3**—*Continued*

$$R_{1,2}^{S} = -\frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + 2J_{IL}^{D}(\omega_I + \omega_L)$$

$$R_{1,4}^{S} = \frac{2}{9\sqrt{2}} J_{IS}^{D}(\omega_I - \omega_S) - \frac{4}{3\sqrt{2}} J_{IS}^{D}(\omega_I) + \frac{4}{3\sqrt{2}} J_{IS}^{D}(\omega_I + \omega_S)$$

$$R_{2,5}^{S} = \frac{2}{9\sqrt{2}} J_{LS}^{D}(\omega_L - \omega_S) - \frac{4}{3\sqrt{2}} J_{LS}^{D}(\omega_L) + \frac{4}{3\sqrt{2}} J_{LS}^{D}(\omega_L + \omega_S)$$

$$R_{3,6}^{S} = \frac{4}{3} K_{IS,LS}^{D,D}(0) + K_{IS,LS}^{D,D}(\omega_S)$$

$$R_{4,5}^{S} = 4K_{IS,LS}^{D,D}(0) - \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + 2J_{IL}^{D}(\omega_I + \omega_L) + 3K_{IS,LS}^{D,D}(\omega_S)$$

$$R_{7,11}^{S} = \frac{\sqrt{2}}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{\sqrt{2}}{9} J_{LS}^{D}(\omega_L - \omega_S) - \frac{2\sqrt{2}}{3} J_{IS}^{D}(\omega_I)$$

$$- \frac{2\sqrt{2}}{3} J_{LS}^{D}(\omega_L) + \frac{2\sqrt{2}}{3} J_{IS}^{D}(\omega_I + \omega_S) + \frac{2\sqrt{2}}{3} J_{LS}^{D}(\omega_L + \omega_S)$$

$$R_{8,9}^{S} = \frac{4}{3} K_{IL,LS}^{D,D}(0) - \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + 6J_{IL}^{D}(\omega_I + \omega_L) + K_{IS,LS}^{D,D}(\omega_S)$$

$$R_{10,11}^{S} = 4K_{IS,LS}^{D,D}(0) + 3K_{IS,LS}^{D,D}(\omega_S)$$

$$R_{1,7}^{S} = -4K_{IL,I}^{D,C}(\omega_I)$$

$$R_{2,7}^{S} = -4K_{IL,L}^{D,C}(\omega_L)$$

$$R_{3,8}^{S} = -\frac{8}{3} K_{IS,S}^{D,C}(0) - 2K_{IS,S}^{D,C}(\omega_S)$$

$$R_{3,9}^{S} = -\frac{8}{3} K_{LS,S}^{D,C}(0) - 2K_{LS,S}^{D,C}(\omega_S)$$

$$R_{4,10}^{S} = -8K_{IS,S}^{D,C}(0) - 6K_{IS,S}^{D,C}(\omega_S)$$

$$R_{4,11}^{S} = -8K_{LS,S}^{D,C}(0) - 4K_{IL,J}^{D,C}(\omega_I) - 6K_{LS,S}^{D,C}(\omega_S)$$

$$R_{5,10}^{S} = -8K_{LS,S}^{D,C}(0) - 6K_{LS,S}^{D,C}(\omega_S)$$

$$R_{5,11}^{S} = -8K_{IS,S}^{D,C}(0) - 4K_{IL,L}^{D,C}(\omega_L) - 6K_{IS,S}^{D,C}(\omega_S)$$

$$R_{6,8}^{S} = -\frac{8}{3} K_{LS,S}^{D,C}(0) - 4K_{IL,I}^{D,C}(\omega_I) - 2K_{LS,S}^{D,C}(\omega_S)$$

$$R_{6,9}^{S} = -\frac{8}{3} K_{IS,S}^{D,C}(0) - 4K_{IL,L}^{D,C}(\omega_L) - 2K_{IS,S}^{D,C}(\omega_S)$$

$$T_{0,0}^{D}(k, l) = -\frac{4}{\sqrt{6}} I_z^k I_z^l,$$

$$T_{0,1}^{D}(k, l) = \frac{1}{\sqrt{6}} I_+^k I_-^l \exp[i(\omega_k - \omega_l)t],$$

$$T_{0,2}^{D}(k, l) = \frac{1}{\sqrt{6}} I_-^k I_+^l \exp[i(\omega_l - \omega_k)t],$$

$$T_{\pm1,0}^{D}(k, l) = \pm I_z^k I_\pm^l \exp[\pm i\omega_l t],$$

$$T_{\pm1,1}^{D}(k, l) = \pm I_\pm^k I_z^l \exp[\pm i\omega_k t],$$

$$T_{\pm2,0}^{D}(k, l) = -I_\pm^k I_\pm^l \exp[\pm i(\omega_k + \omega_l)t], \quad [5a]$$

For the quadrupolar interaction of the spin $k$ we have

$$T_{0,0}^{Q}(k) = \sqrt{\frac{2}{3}} (3(I_z^k)^2 - (I^k)^2),$$

$$T_{\pm1,0}^{Q}(k) = \mp I_\pm^k (2I_z^k \pm 1) \exp[\pm i\omega_k t],$$

$$T_{\pm2,0}^{Q}(k) = I_\pm^k I_\pm^k \exp[\pm i2\omega_k t], \quad [5b]$$

while the CSA terms of the spin $k$ are

$$T_{0,0}^{C}(k) = \frac{4}{\sqrt{6}} I_z^k,$$

$$T_{\pm1,0}^{C}(k) = \mp I_\pm^k \exp[\pm i\omega_k t]. \quad [5c]$$

We will use secular approximation; thus the only terms that produce relaxation are those in which the time dependence of $T_{p,-q}^{\mu}$ cancels that of $T_{p,q}^{\eta}$ in Eq. [4]. Furthermore, AMX is a spin system without degenerate transitions, and therefore the relaxation matrix has a block-diagonal form, i.e., populations are decoupled from the coherences which, in turn, are decoupled from each other.

For the interactions considered in this paper terms $V_q^{\eta}$ in Eq. [2] can be represented as the products of interaction coupling constants $\zeta^{\eta}$ and normalized rank two spherical harmonics $Y_q^{\eta}$ describing the orientation of the principal frame of the interaction $\eta$ with respect to the laboratory frame. The latter is time dependent because of molecular motion. Coupling constants

**TABLE 4**
**Nonzero Relaxation Matrix Elements with Both Spin-1/2 Nuclei ($I$ and $L$) Spin-Locked**

$$R_{1,1}^{IL} = \frac{2}{3} J_{IL}^{D}(0) + \frac{16}{9} J_{IS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{4}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L) + \frac{4}{3} J_{IS}^{D}(\omega_I) + \frac{8}{3} J_{IS}^{D}(\omega_S)$$
$$+ J_{IL}^{D}(\omega_I + \omega_L) + \frac{8}{3} J_{IS}^{D}(\omega_I + \omega_S) + 2J_{I}^{C}(\omega_I)$$

$$R_{2,2}^{IL} = \frac{2}{3} J_{IL}^{D}(0) + \frac{16}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{4}{9} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + \frac{4}{3} J_{LS}^{D}(\omega_L)$$
$$+ \frac{8}{3} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{8}{3} J_{LS}^{D}(\omega_L + \omega_S) + 2J_{L}^{C}(\omega_L)$$

$$R_{3,3}^{IL} = \frac{1}{3} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{3} J_{LS}^{D}(\omega_L - \omega_S) + J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_S) + 2J_{IS}^{D}(\omega_I + \omega_S) + 2J_{LS}^{D}(\omega_L + \omega_S) + 4J_{S}^{Q}(\omega_S) + 16J_{S}^{Q}(2\omega_S) + 4J_{S}^{C}(\omega_S)$$

$$R_{4,4}^{IL} = \frac{2}{3} J_{IL}^{D}(0) + \frac{8}{9} J_{IS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{5}{9} J_{IS}^{D}(\omega_I - \omega_S) + J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L) + \frac{2}{3} J_{IS}^{D}(\omega_I)$$
$$+ \frac{1}{3} J_{IS}^{D}(\omega_S) + 3J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{10}{3} J_{IS}^{D}(\omega_I + \omega_S) + 6J_{LS}^{D}(\omega_L + \omega_S) + 12J_{S}^{Q}(\omega_S) + 2J_{I}^{C}(\omega_I) + 12J_{S}^{C}(\omega_S)$$

$$R_{5,5}^{IL} = \frac{2}{3} J_{IL}^{D}(0) + \frac{8}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + J_{IS}^{D}(\omega_I - \omega_S) + \frac{5}{9} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + 3J_{IS}^{D}(\omega_S)$$
$$+ \frac{2}{3} J_{LS}^{D}(\omega_L) + \frac{1}{3} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + 6J_{IS}^{D}(\omega_I + \omega_S) + \frac{10}{3} J_{LS}^{D}(\omega_L + \omega_S) + 12J_{S}^{Q}(\omega_S) + 2J_{L}^{C}(\omega_L) + 12J_{S}^{C}(\omega_S)$$

$$R_{6,6}^{IL} = \frac{8}{3} J_{IS}^{D}(0) + \frac{8}{3} J_{LS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{3} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{3} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I)$$
$$+ \frac{1}{2} J_{IL}^{D}(\omega_L) + 2J_{IS}^{D}(\omega_I) + J_{IS}^{D}(\omega_S) + 2J_{LS}^{D}(\omega_L) + J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + 2J_{IS}^{D}(\omega_I + \omega_S)$$
$$+ 2J_{LS}^{D}(\omega_L + \omega_S) + 4J_{S}^{Q}(\omega_S) + 16J_{S}^{Q}(2\omega_S) + 2J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L) + 4J_{S}^{C}(\omega_S)$$

$$R_{7,7}^{IL} = \frac{16}{9} J_{IS}^{D}(0) + \frac{16}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{4}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{4}{9} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L)$$
$$+ \frac{4}{3} J_{IS}^{D}(\omega_I) + \frac{8}{3} J_{IS}^{D}(\omega_S) + \frac{4}{3} J_{LS}^{D}(\omega_L) + \frac{8}{3} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{8}{3} J_{IS}^{D}(\omega_I + \omega_S) + \frac{8}{3} J_{LS}^{D}(\omega_L + \omega_S) + 2J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L)$$

$$R_{8,8}^{IL} = \frac{2}{3} J_{IL}^{D}(0) + \frac{8}{3} J_{IS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{3} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{3} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L) + 2J_{IS}^{D}(\omega_I)$$
$$+ J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + 2J_{IS}^{D}(\omega_I + \omega_S) + 2J_{LS}^{D}(\omega_L + \omega_S) + 4J_{S}^{Q}(\omega_S) + 16J_{S}^{Q}(2\omega_S) + 2J_{I}^{C}(\omega_I) + 4J_{S}^{C}(\omega_S)$$

$$R_{9,9}^{IL} = \frac{2}{3} J_{IL}^{D}(0) + \frac{8}{3} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{3} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{3} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + J_{IS}^{D}(\omega_S)$$
$$+ 2J_{LS}^{D}(\omega_L) + J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + 2J_{IS}^{D}(\omega_I + \omega_S) + 2J_{LS}^{D}(\omega_L + \omega_S) + 4J_{S}^{Q}(\omega_S) + 16J_{S}^{Q}(2\omega_S) + 2J_{L}^{C}(\omega_L) + 4J_{S}^{C}(\omega_S)$$
$$R_{10,10}^{IL} = J_{IS}^{D}(\omega_I - \omega_S) + J_{LS}^{D}(\omega_L - \omega_S) + 3J_{IS}^{D}(\omega_S) + 3J_{LS}^{D}(\omega_S) + 6J_{IS}^{D}(\omega_I + \omega_S) + 6J_{LS}^{D}(\omega_L + \omega_S) + 12J_{S}^{Q}(\omega_S) + 12J_{S}^{C}(\omega_S)$$

$$R_{11,11}^{I} = \frac{8}{9} J_{IS}^{D}(0) + \frac{8}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{5}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{5}{9} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I)$$
$$+ \frac{1}{2} J_{IL}^{D}(\omega_L) + \frac{2}{3} J_{IS}^{D}(\omega_I) + \frac{1}{3} J_{IS}^{D}(\omega_S) + \frac{2}{3} J_{LS}^{D}(\omega_L) + \frac{1}{3} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{10}{3} J_{IS}^{D}(\omega_I + \omega_S)$$
$$+ \frac{10}{3} J_{LS}^{D}(\omega_L + \omega_S) + 12J_{S}^{Q}(\omega_S) + 2J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L) + 12J_{S}^{C}(\omega_S)$$

$$R_{1,4}^{IL} = \frac{16}{9\sqrt{2}} J_{IS}^{D}(0) - \frac{2}{9\sqrt{2}} J_{IS}^{D}(\omega_I - \omega_S) + \frac{4}{3\sqrt{2}} J_{IS}^{D}(\omega_I) - \frac{4}{3\sqrt{2}} J_{IS}^{D}(\omega_S) - \frac{4}{3\sqrt{2}} J_{IS}^{D}(\omega_I + \omega_S)$$

$$R_{2,5}^{IL} = \frac{16}{9\sqrt{2}} J_{LS}^{D}(0) - \frac{2}{9\sqrt{2}} J_{LS}^{D}(\omega_L - \omega_S) + \frac{4}{3\sqrt{2}} J_{LS}^{D}(\omega_L) - \frac{4}{3\sqrt{2}} J_{LS}^{D}(\omega_S) - \frac{4}{3\sqrt{2}} J_{LS}^{D}(\omega_L + \omega_S)$$

$$R_{7,11}^{I} = \frac{8\sqrt{2}}{9} J_{IS}^{D}(0) + \frac{8\sqrt{2}}{9} J_{LS}^{D}(0) - \frac{\sqrt{2}}{9} J_{IS}^{D}(\omega_I - \omega_S) - \frac{\sqrt{2}}{9} J_{LS}^{D}(\omega_L - \omega_S) + \frac{2\sqrt{2}}{3} J_{IS}^{D}(\omega_I) - \frac{2\sqrt{2}}{3} J_{IS}^{D}(\omega_S)$$
$$+ \frac{2\sqrt{2}}{3} J_{LS}^{D}(\omega_L) - \frac{2\sqrt{2}}{3} J_{LS}^{D}(\omega_S) - \frac{2\sqrt{2}}{3} J_{IS}^{D}(\omega_I + \omega_S) - \frac{2\sqrt{2}}{3} J_{LS}^{D}(\omega_L + \omega_S)$$

**TABLE 4**—*Continued*

$$R_{1,8}^{IL} = -\frac{16\sqrt{6}}{9} K_{IS,I}^{D,C}(0) - \frac{8}{\sqrt{6}} K_{IS,I}^{D,C}(\omega_I)$$

$$R_{2,9}^{IL} = -\frac{16\sqrt{6}}{9} K_{LS,L}^{D,C}(0) - \frac{8}{\sqrt{6}} K_{LS,L}^{D,C}(\omega_L)$$

$$R_{3,10}^{IL} = \frac{24}{\sqrt{3}} K_{S,S}^{Q,C}(\omega_S)$$

$$R_{4,8}^{IL} = -\frac{16}{3\sqrt{3}} K_{IS,I}^{D,C}(0) - \frac{4}{\sqrt{3}} K_{IS,I}^{D,C}(\omega_I) + \frac{24}{\sqrt{3}} K_{S,S}^{D,C}(\omega_S)$$

$$R_{5,9}^{IL} = -\frac{16}{3\sqrt{3}} K_{LS,L}^{D,C}(0) - \frac{4}{\sqrt{3}} K_{LS,L}^{D,C}(\omega_L) + \frac{24}{\sqrt{3}} K_{S,S}^{D,C}(\omega_S)$$

$$R_{6,7}^{IL} = -\frac{16}{3}\sqrt{\frac{2}{3}} K_{IS,I}^{D,C}(0) - \frac{16}{3}\sqrt{\frac{2}{3}} K_{LS,L}^{D,C}(0) - 4\sqrt{\frac{2}{3}} K_{IS,I}^{D,C}(\omega_I) - 4\sqrt{\frac{2}{3}} K_{LS,L}^{D,C}(\omega_L)$$

$$R_{6,11}^{IL} = -\frac{16}{3\sqrt{3}} K_{IS,I}^{D,C}(0) - \frac{16}{3\sqrt{3}} K_{LS,L}^{D,C}(0) - \frac{4}{\sqrt{3}} K_{IS,I}^{D,C}(\omega_I) - \frac{4}{\sqrt{3}} K_{LS,L}^{D,C}(\omega_L) + \frac{24}{\sqrt{3}} K_{S,S}^{Q,C}(\omega_S)$$

can be found in the review of Werbelow (*9*) and are the following for the dipolar interactions between spins $k$ and $l$:

$$\zeta_{k,l}^D = \sqrt{\frac{6\pi}{5}} \frac{\gamma_k \gamma_l \hbar}{r_{kl}^3}, \qquad [6a]$$

where $\gamma_k$ and $\gamma_l$ are magnetogyric ratios of the nuclei and $\hbar$ is the Planck constant. For quadrupolar relaxation of nucleus $k$ the interaction constant is

$$\zeta_k^Q = \sqrt{\frac{3\pi}{10}} \frac{QCC_k}{2I(2I-1)}, \qquad [6b]$$

where QCC is the quadrupolar coupling constant.

Finally, for the CSA mechanism of nucleus $k$ we have

$$\zeta_k^C = \sqrt{\frac{2\pi}{15}} (\gamma_k B_0 \Delta\sigma_k), \qquad [6c]$$

where $\Delta\sigma$ is the chemical shielding anisotropy.

Magnetization modes in Eq. [3] for the AMX spin system can be constructed as a direct product of irreducible spin operators of zeroth order (*3*) for isolated spin modes (*8–10*). Thus an orthonormal set of magnetization modes for the AMX spin system, where the nuclei A, M, and X have spins $I = 1/2$, $L = 1/2$, and $S = 1$, respectively, is

$$\nu_{1(I)}^1 = \Delta\left\langle \sqrt{\frac{1}{3}} I_z \right\rangle,$$

$$\nu_{1(L)}^2 = \Delta\left\langle \sqrt{\frac{1}{3}} L_z \right\rangle,$$

$$\nu_{1(S)}^3 = \Delta\left\langle \frac{1}{2\sqrt{2}} S_z \right\rangle,$$

$$\nu_{3(IS)}^4 = \left\langle \sqrt{\frac{1}{6}} I_z(3S_z^2 - S^2) \right\rangle,$$

$$\nu_{3(LS)}^5 = \left\langle \sqrt{\frac{1}{6}} L_z(3S_z^2 - S^2) \right\rangle,$$

$$\nu_{3(ILS)}^6 = \sqrt{2}\langle I_z L_z S_z \rangle,$$

$$\nu_{2(IL)}^7 = \frac{2}{\sqrt{3}} \langle I_z L_z \rangle,$$

$$\nu_{2(IS)}^8 = \frac{1}{\sqrt{2}} \langle I_z S_z \rangle,$$

$$\nu_{2(LS)}^9 = \frac{1}{\sqrt{2}} \langle L_z S_z \rangle,$$

$$\nu_{2(S)}^{10} = \left\langle \frac{1}{2\sqrt{6}} (3S_z^2 - S^2) \right\rangle,$$

$$\nu_{4(ILS)}^{11} = \left\langle \sqrt{\frac{2}{3}} I_z L_z(3S_z^2 - S^2) \right\rangle, \qquad [7]$$

where the index $i$ in $\nu_i^k$ corresponds to spin order, and $k$ is the index describing the elements in the relaxation matrix.

Before going to the next section, we must make two assumptions about the conditions that quench the dependence of the relaxation on the strength and the offset of the RF (*25–28*). First, the RF field strength must be small enough, so that the product of the correlation time of the interactions responsible for the relaxation, $\tau_c$, and the RF nutation frequency, $\omega_1$, is much less then unity. This means that the RF field does not interfere with the relaxation processes, and spectral densities at $\omega_1$ can be replaced by spectral densities at zero frequency. The second assumption requires that an ideal spin-lock is achieved. Thus the RF is applied exactly on-resonance and is strong enough to affect all multiplet transitions equally. Under these

<div align="center">

**TABLE 5**

**Nonzero Relaxation Matrix Elements with a Spin-1/2 Nucleus ($L$) and a Spin-1 Nucleus ($S$) Spin-Locked**

</div>

$$R_{1,1}^{LS} = \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{8}{9} J_{IS}^{D}(\omega_I - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{8}{3} J_{IS}^{D}(\omega_I) + 2J_{IL}^{D}(\omega_I + \omega_L) + \frac{16}{3} J_{IS}^{D}(\omega_I + \omega_S) + 4J_{I}^{C}(\omega_I)$$

$$R_{2,2}^{LS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{16}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{4}{9} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + \frac{4}{3} J_{LS}^{D}(\omega_L)$$

$$+ \frac{8}{3} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{8}{3} J_{LS}^{D}(\omega_L + \omega_S) + 2J_{L}^{C}(\omega_L)$$

$$R_{3,3}^{LS} = \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_{S}^{Q}(0) + \frac{8}{3} J_{S}^{C}(0) + \frac{1}{6} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{6} J_{LS}^{D}(\omega_L - \omega_S) + J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S)$$

$$+ J_{LS}^{D}(\omega_L) + \frac{1}{2} J_{LS}^{D}(\omega_S) + J_{IS}^{D}(\omega_I + \omega_S) + J_{LS}^{D}(\omega_L + \omega_S) + 10J_{S}^{Q}(\omega_S) + 4J_{S}^{Q}(2\omega_S) + 2J_{S}^{C}(\omega_S)$$

$$R_{4,4}^{LS} = 2J_{IS}^{D}(0) + 2J_{LS}^{D}(0) + 8J_{S}^{C}(0) + \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{5}{18} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I)$$

$$+ \frac{1}{3} J_{IS}^{D}(\omega_I) + \frac{3}{2} J_{IS}^{D}(\omega_S) + 3J_{LS}^{D}(\omega_L) + \frac{3}{2} J_{LS}^{D}(\omega_S) + 2J_{IL}^{D}(\omega_I + \omega_L) + \frac{5}{3} J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ 3J_{LS}^{D}(\omega_L + \omega_S) + 6J_{S}^{Q}(\omega_S) + 6J_{S}^{Q}(2\omega_S) + 4J_{I}^{C}(\omega_I) + 6J_{S}^{C}(\omega_S)$$

$$R_{5,5}^{LS} = \frac{2}{3} J_{IL}^{D}(0) + 2J_{IS}^{D}(0) + \frac{2}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + 8J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{2} J_{IS}^{D}(\omega_I - \omega_S) + \frac{7}{18} J_{LS}^{D}(\omega_L - \omega_S)$$

$$+ J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + 3J_{IS}^{D}(\omega_I) + \frac{3}{2} J_{IS}^{D}(\omega_S) + \frac{5}{3} J_{LS}^{D}(\omega_L) + \frac{5}{6} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + 3J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ \frac{7}{3} J_{LS}^{D}(\omega_L + \omega_S) + 6J_{S}^{Q}(\omega_S) + 6J_{S}^{Q}(2\omega_S) + 2J_{L}^{C}(\omega_L) + 6J_{S}^{C}(\omega_S)$$

$$R_{6,6}^{LS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_{S}^{Q}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{8}{3} J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{5}{6} J_{IS}^{D}(\omega_I - \omega_S)$$

$$+ \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_L) + J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{5}{2} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L)$$

$$+ 5J_{IS}^{D}(\omega_I + \omega_S) + 3J_{LS}^{D}(\omega_L + \omega_S) + 10J_{S}^{Q}(\omega_S) + 4J_{S}^{Q}(2\omega_S) + 4J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L) + 2J_{S}^{C}(\omega_S)$$

$$R_{7,7}^{LS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{16}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{8}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{4}{9} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_L) + \frac{8}{3} J_{IS}^{D}(\omega_I)$$

$$+ \frac{4}{3} J_{LS}^{D}(\omega_L) + \frac{8}{3} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{16}{3} J_{IS}^{D}(\omega_I + \omega_S) + \frac{8}{3} J_{LS}^{D}(\omega_L + \omega_S) + 4J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L)$$

$$R_{8,8}^{LS} = \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_{S}^{Q}(0) + \frac{8}{3} J_{S}^{C}(0) + \frac{1}{3} J_{IL}^{D}(\omega_I - \omega_L) + \frac{5}{6} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{6} J_{LS}^{D}(\omega_L - \omega_S)$$

$$+ J_{IL}^{D}(\omega_I) + J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{1}{2} J_{LS}^{D}(\omega_S) + 2J_{IL}^{D}(\omega_I + \omega_L) + 5J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ J_{LS}^{D}(\omega_L + \omega_S) + 10J_{S}^{Q}(\omega_S) + 4J_{S}^{Q}(2\omega_S) + 4J_{I}^{C}(\omega_I) + 2J_{S}^{C}(\omega_S)$$

$$R_{9,9}^{LS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_{S}^{Q}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{8}{3} J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{6} J_{IS}^{D}(\omega_I - \omega_S)$$

$$+ \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{5}{2} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L)$$

$$+ J_{IS}^{D}(\omega_I + \omega_S) + 3J_{LS}^{D}(\omega_L + \omega_S) + 10J_{S}^{Q}(\omega_S) + 4J_{S}^{Q}(2\omega_S) + 2J_{L}^{C}(\omega_L) + 2J_{S}^{C}(\omega_S)$$

$$R_{10,10}^{LS} = 2J_{IS}^{D}(0) + 2J_{LS}^{D}(0) + 8J_{S}^{C}(0) + \frac{1}{2} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + 3J_{IS}^{D}(\omega_I) + \frac{3}{2} J_{IS}^{D}(\omega_S) + 3J_{LS}^{D}(\omega_L)$$

$$+ \frac{3}{2} J_{LS}^{D}(\omega_S) + 3J_{IS}^{D}(\omega_I + \omega_S) + 3J_{LS}^{D}(\omega_L + \omega_S) + 6J_{S}^{Q}(\omega_S) + 6J_{S}^{Q}(2\omega_S) + 6J_{S}^{C}(\omega_S)$$

$$R_{11,11}^{LS} = \frac{2}{3} J_{IL}^{D}(0) + 2J_{IS}^{D}(0) + \frac{2}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + 8J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{5}{18} J_{IS}^{D}(\omega_I - \omega_S)$$

$$+ \frac{7}{18} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_L) + \frac{1}{3} J_{IS}^{D}(\omega_I) + \frac{3}{2} J_{IS}^{D}(\omega_S) + \frac{5}{3} J_{LS}^{D}(\omega_L) + \frac{5}{6} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L)$$

$$+ \frac{5}{3} J_{IS}^{D}(\omega_I + \omega_S) + \frac{7}{3} J_{LS}^{D}(\omega_L + \omega_S) + 6J_{S}^{Q}(\omega_S) + 6J_{S}^{Q}(2\omega_S) + 4J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L) + 6J_{S}^{C}(\omega_S)$$

<div style="text-align:center"><strong>TABLE 5</strong>—<em>Continued</em></div>

$$R_{1,4}^{LS} = -\frac{4}{9\sqrt{2}} J_{IS}^{D}(\omega_I - \omega_S) + \frac{8}{3\sqrt{2}} J_{IS}^{D}(\omega_I) - \frac{8}{3\sqrt{2}} J_{IS}^{D}(\omega_I + \omega_S)$$

$$R_{2,5}^{LS} = -\frac{8}{9\sqrt{2}} J_{LS}^{D}(0) + \frac{1}{9\sqrt{2}} J_{LS}^{D}(\omega_L - \omega_S) - \frac{2}{3\sqrt{2}} J_{LS}^{D}(\omega_L) + \frac{2}{3\sqrt{2}} J_{LS}^{D}(\omega_S) + \frac{2}{3\sqrt{2}} J_{LS}^{D}(\omega_L + \omega_S)$$

$$R_{7,11}^{LS} = -\frac{8}{9\sqrt{2}} J_{LS}^{D}(0) + \frac{\sqrt{2}}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{\sqrt{2}}{18} J_{LS}^{D}(\omega_L - \omega_S) - \frac{2\sqrt{2}}{3} J_{IS}^{D}(\omega_I) - \frac{\sqrt{2}}{3} J_{LS}^{D}(\omega_L)$$

$$+ \frac{\sqrt{2}}{3} J_{LS}^{D}(\omega_S) + \frac{2\sqrt{2}}{3} J_{IS}^{D}(\omega_I + \omega_S) + \frac{\sqrt{2}}{3} J_{LS}^{D}(\omega_L + \omega_S)$$

$$R_{2,7}^{LS} = -\frac{8}{3} K_{IL,L}^{D,C}(0) - 2K_{IL,L}^{D,C}(\omega_L)$$

$$R_{3,8}^{LS} = -\frac{8}{3} K_{IS,S}^{D,C}(0) - 2K_{IS,S}^{D,C}(\omega_S)$$

$$R_{4,10}^{LS} = -8K_{IS,S}^{D,C}(0) - 6K_{IS,S}^{D,C}(\omega_S)$$

$$R_{5,11}^{LS} = -\frac{8}{3} K_{IL,L}^{D,C}(0) - 8K_{IS,S}^{D,C}(0) - 2K_{IL,L}^{D,C}(\omega_L) - 6K_{IS,S}^{D,C}(\omega_S)$$

$$R_{6,9}^{LS} = -\frac{8}{3} K_{IL,L}^{D,C}(0) - \frac{8}{3} K_{IS,S}^{D,C}(0) - 2K_{IL,L}^{D,C}(\omega_L) - 2K_{IS,S}^{D,C}(\omega_S)$$

spin-locking conditions, the quantization axis is directed along the $B_1$ field applied parallel to the $x$ axis (15), and therefore $z$ in Eq. [7] should be changed to $x$.

## RESULTS AND DISCUSSION

The results below were obtained with the help of the C++ NMR library GAMMA (29). Since the relaxation matrices are symmetric, we present only diagonal and above-diagonal elements.

To estimate the strength of different relaxation mechanisms we take the spin system containing amide proton, nitrogen-15, and $\alpha$ deuteron, with the spins denoted by $I$, $L$, and $S$, respectively. This system can be found in $^{15}$N- and $^{2}$H-labeled proteins in $H_2O$ solution, after amide deuterons have been exchanged by protons. Typical $r_{HN} = 104$, $r_{ND} = 207$, $r_{HN} = 260$ pm, $\Delta\sigma_H = 13$, $\Delta\sigma_N = -157$, $\Delta\sigma_D = 10$ ppm, QCC$^D$ = 180 kHz. With these values one obtains for 500 MHz magnetic field $\zeta_{HN}^{D} = 132.0 \times 10^3$, $\zeta_{ND}^{D} = 2.6 \times 10^3$, $\zeta_{HD}^{D} = 5.6 \times 10^3$, $\zeta_{H}^{C} = 26.5 \times 10^3$, $\zeta_{N}^{C} = 32.4 \times 10^3$, $\zeta_{D}^{C} = 3.1 \times 10^3$, $\zeta_{D}^{Q} = 549.0 \times 10^3$ s$^{-1}$. Thus one can see that in this spin system the quadrupolar interaction is clearly dominating for deuterons. The strength of the dipolar interaction between amide proton and nitrogen is comparable to CSA-type interactions on respective nuclei, which is the foundation of the TROSY effect (30).

For isotropic molecular reorientation $J^{\mu,\lambda}(\omega)$ of Eq. [4] are described by (23)

$$J^{\mu,\lambda}(\omega) = \zeta^{\mu}\zeta^{\lambda} \frac{1}{8\pi} (3\cos^2\theta - 1) \frac{\tau_c}{1 + (\tau_c\omega)^2}, \quad [8]$$

where $\theta$ is the angle between principal axes of the correspond-

ing interactions. Thus, if the correlation time, $\tau_c$, is long, the last factor in the spectral density function favors the function sampled at zero frequency. As it will be shown, under certain spin-locking conditions, spectral density functions associated with strong interactions occur in the relaxation expressions taken at nonzero frequencies, while weaker interactions are sampled at zero frequency. This fact makes corresponding relaxation mechanisms comparable, even if their interaction constants are very different. Thus, for example, in the spin system described above, disregarding the $\theta$ dependence and in the slow motion regime, $K_{HN,HD}^{D,D}(0)/J_D^{Q}(\omega_D) \cong (\omega_D\tau_c)^2 \zeta_{HD}^{D}\zeta_{HN}^{D}/\zeta_D^{Q}\zeta_D^{Q} \cong 0.05$, for a 60 kDa protein at 500 MHz magnetic field, and this ratio would be 0.2 at 1000 MHz.

*Case 1. No RF field is applied.* Nonzero elements of the relaxation matrix for magnetization modes are given in Table 1. If only quadrupolar and dipolar relaxation mechanisms are taken into account the relaxation matrix can be separated into two blocks. These blocks connect magnetization modes differing in spin order by ±2 or 0, so that parity is conserved. The presence of dipolar–CSA and quadrupolar–CSA interactions couples modes differing by ±1 in spin order and destroys the block diagonal form of the relaxation matrix. The diagonal of the relaxation matrix in only affected by auto-correlation interactions, and cross-correlation terms are sampled at the frequencies common to the two interfering interactions. These are very natural results, as has been shown by Bull (15) in general terms.

It is also important to note the absence of spectral densities at zero frequency. For large biomolecules, the rotational correlation time becomes rather long (we assume, for simplicity, isotropic rigid body reorientation with a correlation time $\tau_c$) so that the slow motional regime, $\omega_k\tau_c \gg 1$, is reached. In this case spectral densities sampled at zero frequency are dominat-

### TABLE 6
### Nonzero Relaxation Matrix Elements with All Three Nuclei Spin-Locked

$$R_{1,1}^{ILS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{16}{9} J_{IS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{4}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L) + \frac{4}{3} J_{IS}^{D}(\omega_I)$$

$$+ \frac{8}{3} J_{IS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{8}{3} J_{IS}^{D}(\omega_I + \omega_S) + 2J_{I}^{C}(\omega_I)$$

$$R_{2,2}^{ILS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{16}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{4}{9} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + \frac{4}{3} J_{LS}^{D}(\omega_L)$$

$$+ \frac{8}{3} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{8}{3} J_{LS}^{D}(\omega_L + \omega_S) + 2J_{L}^{C}(\omega_L)$$

$$R_{3,3}^{ILS} = \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_{S}^{Q}(0) + \frac{8}{3} J_{S}^{C}(0) + \frac{1}{6} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{6} J_{LS}^{D}(\omega_L - \omega_S) + J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L)$$

$$+ \frac{1}{2} J_{LS}^{D}(\omega_S) + J_{IS}^{D}(\omega_I + \omega_S) + J_{LS}^{D}(\omega_L + \omega_S) + 10J_{S}^{Q}(\omega_S) + 4J_{S}^{Q}(2\omega_S) + 2J_{S}^{C}(\omega_S)$$

$$R_{4,4}^{ILS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{2}{9} J_{IS}^{D}(0) + 2J_{LS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + 8J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{7}{18} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L) + \frac{5}{3} J_{IS}^{D}(\omega_I)$$

$$+ \frac{5}{6} J_{IS}^{D}(\omega_S) + 3J_{LS}^{D}(\omega_L) + \frac{3}{2} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{7}{3} J_{IS}^{D}(\omega_I + \omega_S) + 3J_{LS}^{D}(\omega_L + \omega_S) + 6J_{S}^{Q}(\omega_S) + 6J_{S}^{Q}(2\omega_S) + 2J_{I}^{C}(\omega_I) + 6J_{S}^{C}(\omega_S)$$

$$R_{5,5}^{ILS} = \frac{2}{3} J_{IL}^{D}(0) + 2J_{IS}^{D}(0) + \frac{2}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{L}^{C}(0) + 8J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{2} J_{IS}^{D}(\omega_I - \omega_S) + \frac{7}{18} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + 3J_{IS}^{D}(\omega_I)$$

$$+ \frac{3}{2} J_{IS}^{D}(\omega_S) + \frac{5}{3} J_{LS}^{D}(\omega_L) + \frac{5}{6} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + 3J_{IS}^{D}(\omega_I + \omega_S) + \frac{7}{3} J_{LS}^{D}(\omega_L + \omega_S) + 6J_{S}^{Q}(\omega_S) + 6J_{S}^{Q}(2\omega_S) + 2J_{L}^{C}(\omega_L) + 6J_{S}^{C}(\omega_S)$$

$$R_{6,6}^{ILS} = \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_{S}^{Q}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{8}{3} J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{2} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S)$$

$$+ \frac{1}{2} J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + J_{IS}^{D}(\omega_I) + \frac{5}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{5}{2} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + 3J_{IS}^{D}(\omega_I + \omega_S) + 3J_{LS}^{D}(\omega_L + \omega_S)$$

$$+ 10J_{S}^{Q}(\omega_S) + 4J_{S}^{Q}(2\omega_S) + 2J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L) + 2J_{S}^{C}(\omega_S)$$

$$R_{7,7}^{ILS} = \frac{16}{9} J_{IS}^{D}(0) + \frac{16}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{4}{9} J_{IS}^{D}(\omega_I - \omega_S) + \frac{4}{9} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + \frac{4}{3} J_{IS}^{D}(\omega_I)$$

$$+ \frac{8}{3} J_{IS}^{D}(\omega_S) + \frac{4}{3} J_{LS}^{D}(\omega_L) + \frac{8}{3} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{8}{3} J_{IS}^{D}(\omega_I + \omega_S) + \frac{8}{3} J_{LS}^{D}(\omega_L + \omega_S) + 2J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L)$$

$$R_{8,8}^{ILS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_{S}^{Q}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{8}{3} J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{2} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{6} J_{LS}^{D}(\omega_L - \omega_S) + \frac{1}{2} J_{IL}^{D}(\omega_I) + J_{IL}^{D}(\omega_L)$$

$$+ J_{IS}^{D}(\omega_I) + \frac{5}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{1}{2} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + 3J_{IS}^{D}(\omega_I + \omega_S) + J_{LS}^{D}(\omega_L + \omega_S) + 10J_{S}^{Q}(\omega_S) + 4J_{S}^{Q}(2\omega_S) + 2J_{I}^{C}(\omega_L) + 2J_{S}^{C}(\omega_S)$$

$$R_{9,9}^{ILS} = \frac{2}{3} J_{IL}^{D}(0) + \frac{2}{3} J_{IS}^{D}(0) + \frac{2}{3} J_{LS}^{D}(0) + 6J_{S}^{Q}(0) + \frac{8}{3} J_{L}^{C}(0) + \frac{8}{3} J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{1}{6} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L)$$

$$+ J_{IS}^{D}(\omega_I) + \frac{1}{2} J_{IS}^{D}(\omega_S) + J_{LS}^{D}(\omega_L) + \frac{5}{2} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + J_{IS}^{D}(\omega_I + \omega_S) + 3J_{LS}^{D}(\omega_L + \omega_S) + 10J_{S}^{Q}(\omega_S) + 4J_{S}^{Q}(2\omega_S) + 2J_{L}^{C}(\omega_L) + 2J_{S}^{C}(\omega_S)$$

$$R_{10,10}^{ILS} = 2J_{IS}^{D}(0) + 2J_{LS}^{D}(0) + 8J_{S}^{C}(0) + \frac{1}{2} J_{IS}^{D}(\omega_I - \omega_S) + \frac{1}{2} J_{LS}^{D}(\omega_L - \omega_S) + 3J_{IS}^{D}(\omega_I) + \frac{3}{2} J_{IS}^{D}(\omega_S) + 3J_{LS}^{D}(\omega_L)$$

$$+ \frac{3}{2} J_{LS}^{D}(\omega_S) + 3J_{IS}^{D}(\omega_I + \omega_S) + 3J_{LS}^{D}(\omega_L + \omega_S) + 6J_{S}^{Q}(\omega_S) + 6J_{S}^{Q}(2\omega_S) + 6J_{S}^{C}(\omega_S)$$

$$R_{11,11}^{ILS} = \frac{2}{9} J_{IS}^{D}(0) + \frac{2}{9} J_{LS}^{D}(0) + \frac{8}{3} J_{I}^{C}(0) + \frac{8}{3} J_{L}^{C}(0) + 8J_{S}^{C}(0) + \frac{1}{6} J_{IL}^{D}(\omega_I - \omega_L) + \frac{7}{18} J_{IS}^{D}(\omega_I - \omega_S) + \frac{7}{18} J_{LS}^{D}(\omega_L - \omega_S)$$

$$+ \frac{1}{2} J_{IL}^{D}(\omega_I) + \frac{1}{2} J_{IL}^{D}(\omega_L) + \frac{5}{3} J_{IS}^{D}(\omega_I) + \frac{5}{6} J_{IS}^{D}(\omega_S) + \frac{5}{3} J_{LS}^{D}(\omega_L) + \frac{5}{6} J_{LS}^{D}(\omega_S) + J_{IL}^{D}(\omega_I + \omega_L) + \frac{7}{3} J_{IS}^{D}(\omega_I + \omega_S)$$

$$+ \frac{7}{3} J_{LS}^{D}(\omega_L + \omega_S) + 6J_{S}^{Q}(\omega_S) + 6J_{S}^{Q}(2\omega_S) + 4J_{I}^{C}(\omega_I) + 2J_{L}^{C}(\omega_L) + 6J_{S}^{C}(\omega_S)$$

**TABLE 6**—*Continued*

$$R_{1,4}^{ILS} = -\frac{8}{9\sqrt{2}} J_{IS}^D(\omega_I - \omega_S) + \frac{1}{9\sqrt{2}} J_{IS}^D(\omega_I - \omega_S) - \frac{2}{3\sqrt{2}} J_{IS}^D(\omega_I) + \frac{2}{3\sqrt{2}} J_{IS}^D(\omega_S) + \frac{2}{3\sqrt{2}} J_{IS}^D(\omega_I + \omega_S)$$

$$R_{2,5}^{ILS} = -\frac{8}{9\sqrt{2}} J_{LS}^D(\omega_L - \omega_S) + \frac{1}{9\sqrt{2}} J_{LS}^D(\omega_L - \omega_S) - \frac{2}{3\sqrt{2}} J_{LS}^D(\omega_L) + \frac{2}{3\sqrt{2}} J_{LS}^D(\omega_S) + \frac{2}{3\sqrt{2}} J_{LS}^D(\omega_L + \omega_S)$$

$$R_{7,11}^{LS} = -\frac{8}{9\sqrt{2}} J_{IS}^D(0) - \frac{8}{9\sqrt{2}} J_{LS}^D(0) + \frac{\sqrt{2}}{18} J_{IS}^D(\omega_I - \omega_S) + \frac{\sqrt{2}}{18} J_{LS}^D(\omega_L - \omega_S) - \frac{\sqrt{2}}{3} J_{IS}^D(\omega_I) + \frac{\sqrt{2}}{3} J_{IS}^D(\omega_S)$$

$$- \frac{\sqrt{2}}{3} J_{LS}^D(\omega_L) + \frac{\sqrt{2}}{3} J_{LS}^D(\omega_S) + \frac{\sqrt{2}}{3} J_{IS}^D(\omega_I + \omega_S) + \frac{\sqrt{2}}{3} J_{LS}^D(\omega_L + \omega_S)$$

ing as discussed above. Thus, in the absence of an applied RF field, relaxation-driven transitions may in principle become infinitely slow. One way to overcome this problem is to measure relaxation in the presence of an RF field.

*Case 2. I spins are locked by RF field.* In the presence of spin-locking radiofrequency field we have contributions from the spectral densities sampled at zero frequency for interactions associated with the locked nuclei. Therefore, in Table 2 and below, we put these spectral densities, dominant for the slow-motion regime, first in the matrix element expressions.

We can see in Table 2 that imposing spin-lock on one of the nuclei creates a "selection rule" on cross-correlation terms: only contributions from dipolar cross-correlation associated with the locked nuclei are nonzero. Therefore, only the $K_{IL,IS}^{D,D}$ term is important. Moreover, the relevant dipolar–quadrupolar cross-correlations $K_{IS,S}^{D,Q}$ are now also quenched. It is important to notice, however, the presence of $K_{IL,I}^{D,C}$, $K_{IS,I}^{D,C}$, $K_{LS,L}^{D,C}$, and $K_{LS,S}^{D,C}$ terms. The first two spectral densities have contributions at zero frequency. These elements destroy the block-diagonal form of the relaxation matrix and are responsible for coupling magnetization modes of different parity. As it was mentioned above, in the spin system used as an example, the term $K_{HN,HD}^{D,D}(0)$ may destroy initial monoexponential relaxation, of the modes $\nu_{3(IS)}^4$, $\nu_{3(ILS)}^6$, $\nu_{2(IS)}^8$, and $\nu_{4(ILS)}^{11}$, dominated by deuteron quadrupolar mechanism in the case without application of the RF field.

*Case 3. S spins are locked by RF field.* The results when quadrupolar spins, $S$, are locked are summarized in Table 3.

The results are essentially the same as in case 2, except for the important contributions from $J_S^Q$ sampled at zero frequency in Table 3, and the absence of any dipolar–quadrupolar cross-correlation terms. Therefore, in the example spin system, all quadrupolar modes will relax by single exponential decay in the initial period of the relaxation process. This, in fact, holds for all the cases when deuterium nuclei are locked.

*Case 4 and 5. I, L and L, S spins, respectively, are locked by RF field.* Now let us consider the case when two nuclei in this spin system are locked. Tables 4 and 5 show the results when two dipolar spins $I$ and $L$, and dipolar and quadrupolar spins $L$ and $S$, respectively, are locked.

We note first that there are no dipolar–dipolar and dipolar–quadrupolar cross-correlations left. Second, the dipolar–CSA cross-correlation terms disappear only when the two relevant spins are locked. Thus, $K_{IL,I}^{D,C}$ and $K_{IL,L}^{D,C}$, and $K_{LS,L}^{D,C}$ and $K_{LS,S}^{D,C}$ are absent in Tables 4 and 5, respectively.

*Case 6. I, L, and S spins are locked by RF field.* Finally, Table 6 represents the relaxation matrix when all three spins ($I$, $L$, and $S$) are spin-locked. Now, all cross-correlation pathways are blocked, and the relaxation network is considerably simplified. There are, in fact, only three off-diagonal terms left. At the same time all possible auto-correlation spectral densities are sampled at zero frequency. Dominating relaxation matrix elements are situated on the diagonal, except for $J_{IS}^D(0)$, which couples modes $\nu_{2(IL)}^7$ and $\nu_{4(ILS)}^{11}$.

## CONCLUSION

We have considered the relaxation network in a multipolar AMX spin system under application of a spin-locking RF field. The results are particularly important for studies of proteins, where fractional $^2$H enrichment is used. This spin system is also the simplest one that involves all possible kinds of dipolar, quadrupolar and CSA auto- and cross-correlation terms.

Our results show that application of an ideal spin-locking field allows one to sample spectral densities at zero frequency for the interactions associated with the locked nuclei. These spectra densities are dominant in the absence of fast motions, a situation which can be reached for large biomolecules such as proteins, or in viscous media. Under certain conditions, spin-

**TABLE 7**
**Blocked Cross-Correlation Interactions under Different Spin-Locking Conditions**

| RF conditions | Blocked interactions |
|---|---|
| No RF field applied | No cross-correlation terms are blocked |
| $I$ spins are locked | $K_{IL,LS}^{D,D}$, $K_{IS,LS}^{D,D}$, $K_{IS,S}^{D,Q}$, $K_{IL,L}^{D,C}$, $K_{IS,S}^{D,C}$ |
| $S$ spins are locked | $K_{IL,IS}^{D,D}$, $K_{IL,LS}^{D,D}$, $K_{IS,S}^{D,Q}$, $K_{LS,S}^{D,Q}$, $K_{IS,I}^{D,C}$, $K_{LS,L}^{D,C}$, $K_{S,S}^{Q,C}$ |
| $I$ and $L$ spins are locked | All $K^{D,D}$, all $K^{D,Q}$, $K_{IL,J}^{D,C}$, $K_{IL,L}^{D,C}$, $K_{IS,S}^{D,C}$, $K_{LS,S}^{D,C}$ |
| $L$ and $S$ spins are locked | All $K^{D,D}$, all $K^{D,Q}$, $K_{LS,L}^{D,C}$, $K_{LS,S}^{D,C}$, $K_{IS,I}^{D,C}$, $K_{IL,I}^{D,C}$, $K_{S,S}^{Q,C}$ |
| $I$, $L$ and $S$ spins are locked | All cross-correlation pathways are blocked |

locking can enhance the relative importance of the weak interactions. The spin-locking field also quenches certain cross-correlation terms. This is summarized in the Table 7. Thus, there exist only dipolar–dipolar cross-correlation terms associated with locked nuclei. Moreover, the application of an RF field on one of the relevant nuclei blocks both dipolar–quadrupolar and quadrupolar–CSA cross correlations, whereas quenching of the dipolar–CSA terms requires spin-locking of both nuclei.

## ACKNOWLEDGMENTS

## REFERENCES

1. R. Tycko (Ed.), "Nuclear Magnetic Resonance Probes of Molecular Dynamics," Kluwer Academic, Dordrecht/Norwell, MA (1994).

2. J. Kowalewski, *Ann. Repts. NMR Spectrosc.* **22,** 309 (1990); **23,** 289 (1991).

3. N. Müller, G. Bodenhausen, and R. R. Ernst, *J. Magn. Reson.* **75,** 297 (1987).

4. S. Wimperis and G. Bodenhausen, *Mol. Phys.* **66,** 897 (1989).

5. D. Neuhaus and M. P. Williamson, "The Nuclear Overhauser Effect in Structural and Conformational Analysis," VCH, New York (1989).

6. A. A. Bothner-By, R. L. Stephens, J. M. Lee, C. D. Warren, and R. W. Jeanloz, *J. Am. Chem. Soc.* **106,** 811 (1984).

7. A. Bax and D. G. Davis, *J. Magn. Reson.* **63,** 207 (1985).

8. L. G. Werbelow, D. A. Ikenberry, and G. Pouzard, *J. Magn. Reson.* **51,** 409 (1983).

9. L. G. Werbelow, *In* "Nuclear Magnetic Resonance Probes of Molecular Dynamics," p. 223, Kluwer Academic, Dordrecht/Norwell, MA (1994).

10. A. Kaikkonen and J. Kowalewski, *J. Magn. Reson.* **141,** 326 (1999).

11. J. Boyd, U. Hommel, and I. D. Campbell, *Chem. Phys. Lett.* **175,** 477 (1990).

12. J. Cavanagh, W. J. Fairbrother, A. G. Palmer III, and N. J. Skelton, "Protein NMR Spectroscopy," Academic Press, San Diego (1996).

13. J. N. S. Evans, "Biomolecular NMR Spectroscopy," Oxford University Press, Oxford (1995).

14. P. E. Fagerness, D. M. Grant, K. F. Kuhlmann, C. L. Mayne, and R. B. Parry, *J. Chem. Phys.* **63,** 2524 (1975).

15. T. E. Bull, *Prog. NMR Spectrosc.* **24,** 377 (1992).

16. R. Brüschweiler, C. Griesinger, and R. R. Ernst, *J. Am. Chem. Soc.* **111,** 8034 (1989).

17. R. Brüschweiler and R. R. Ernst, *J. Chem. Phys.* **96,** 1758 (1992).

18. I. Burghardt, R. Konrat, and G. Bodenhausen, *Mol. Phys.* **75,** 467 (1992).

19. C. Dalvit, *J. Magn. Reson.* **95,** 410 (1991).

20. V. Varma, N. D. Kurur, and G. Bodenhausen, *J. Magn. Reson. A* **118,** 64 (1996).

21. M. Ernst and R. R. Ernst, *J. Magn. Reson. A* **110,** 202 (1994).

22. R. R. Ernst, G. Bodenhausen, and A. Wokaun, "Principles of Nuclear Magnetic Resonance in One and Two Dimensions," Clarendon Press, Oxford (1987).

23. D. E. Woessner, "Encyclopedia of Nuclear Magnetic Resonance," Wiley, Chichester, p. 1068 (1996).

24. L. G. Werbelow and J. Kowalewski, *J. Chem. Phys.* **107,** 2775 (1997).

25. A. Abragam, "The Principles of Nuclear Magnetism," Clarendon Press, Oxford (1961).

26. N. R. Skrynnikov, S. F. Lienin, R. Brüschweiler, and R. R. Ernst, *J. Chem. Phys.* **108,** 7662 (1998).

27. C. P. Slichter, "Principles of Magnetic Resonance," Springer-Verlag, Berlin/New York (1978).

28. S. Grzesiek and A. Bax, *J. Am. Chem. Soc.* **116,** 10,196 (1994).

29. S. A. Smith, T. O. Levante, B. H. Meier, and R. R. Ernst, *J. Magn. Reson.* **106,** 75 (1994).

30. K. Pervushin, R. Riek, G. Wider, and K. Wüthrich, *Proc. Natl. Acad. Sci. USA* **94,** 12,366 (1997).